Problems

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before June 15, 2016

- **5397:** *Proposed by Kenneth Korbin, New York, NY*

  Solve the equation $\sqrt[3]{x+9}=\sqrt{3}+\sqrt[3]{x-9}$ with $x>9$.

- **5398:** *Proposed by D. M. Bătinetu-Giurgiu, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania*

  If $(2n-1)!!=1\cdot 3\cdot 5\ldots(2n-1)$, then evaluate

$$\lim_{n\to\infty}\left(\frac{\sqrt[n+1]{(n+1)!(2n+1)!!}}{n+1}-\frac{\sqrt[n]{n!(2n-1)!!}}{n}\right).$$

- **5399:** *Proposed by Ángel Plaza, University of Las Palmas de Gran Canaria, Spain*

  Let a,b,c be positive real numbers. Prove that

$$\sum_{cyclic}\frac{2a+2b}{\sqrt{6a^2+4ab+6b^2}}\leq 3.$$

5400: *Proposed by Arkady Alt, San Jose, CA*

Prove the inequality

$$\frac{a^2}{m_a}+\frac{b^2}{m_b}+\frac{c^2}{m_c}\leq 12\,(2R-3r),$$

where a,b,c and m_a,m_b,m_c are respectively sides and medians of $\triangle ABC$, with circumradius R and inradius r.

- **5401:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

  Let a,b,c be three positive real numbers such that $a^2+b^2+c^2=3$. Prove that

$$\frac{b^{-1}}{(4\sqrt{a}+3\sqrt{b})^2}+\frac{c^{-1}}{(4\sqrt{b}+3\sqrt{c})^2}+\frac{a^{-1}}{(4\sqrt{c}+3\sqrt{a})^2}\geq\frac{3}{49}.$$